A trimagic square with three pairs of 3-equivalent 4-tuples

Walter Trump, 2018-03-08

The trimagic square 9a has 3 pairs of 3-equivalent 4-tuples and each of them has 4 diagonal entries.
For each pair of 4-tuples there are 2 transformations which lead to a new axially symmetric trimagic square.
We show this for columns 2 and 11 with the blue (cyan) 4-tuples:

transformation i: Identity (leave the square as it is)

transformation r: Swap the 8-tuples which are in the two columns beside the 4-tuples.
(Interchange entries 67, 94, 24, 143, 2, 121, 51, 78 with the entries 63, 138, 131, 76, 69, 14, 7, 82.)

transformation s: Swap the 4-tuples of column 2 and 11 (Interchange the 33, 104, 41, 112 with 74, 22, 123, 71).
Now the diagonals are no longer trimagic, because 74 and 112 are in the descending diagonal instead of 33 and 71. Therefore we additionally have to interchange row 2 with row 11.

transformation t: Apply transformation b and c.

First I thought we would obtain 3^3 = 27 essentially different trimagic squares, but Holger Danielsson informed me that the squares 9b and 9r can be transformed into each other by swapping row 2 and 11 and swapping column 2 and 11. Thus the squares are not essentially different. The same is true for 9a and 9t. This means: s and t are obsolete.

9a (= 9a after transformation i)

12	67	53	137	6	44	91	109	46	114	63	**128**
96	**33**	124	39	47	127	52	38	113	1	**74**	126
102	104	**34**	136	48	5	35	85	135	**83**	22	81
86	94	72	**55**	20	79	4	29	**45**	129	138	119
56	24	134	88	**115**	70	105	**13**	15	77	131	42
3	143	118	50	84	**117**	**23**	25	65	58	76	108
142	2	27	95	61	**28**	**122**	120	80	87	69	37
89	121	11	57	**30**	75	40	**132**	130	68	14	103
59	51	73	**90**	125	66	141	116	**100**	16	7	26
43	41	**111**	9	97	140	110	60	10	**62**	123	64
49	**112**	21	106	98	18	93	107	32	144	**71**	19
133	78	92	8	139	101	54	36	99	31	82	**17**

9b (= 9a after transformation r)

12	**63**	53	137	6	44	91	109	46	114	**67**	**128**
96	**33**	124	39	47	127	52	38	113	1	**74**	126
102	104	**34**	136	48	5	35	85	135	**83**	22	81
86	**138**	72	**55**	20	79	4	29	**45**	129	**94**	119
56	**131**	134	88	**115**	70	105	**13**	15	77	**24**	42
3	**76**	118	50	84	**117**	**23**	25	65	58	**143**	108
142	**69**	27	95	61	**28**	**122**	120	80	87	**2**	37
89	**14**	11	57	**30**	75	40	**132**	130	68	**121**	103
59	**7**	73	**90**	125	66	141	116	**100**	16	**51**	26
43	41	**111**	9	97	140	110	60	10	**62**	123	64
49	**112**	21	106	98	18	93	107	32	144	**71**	19
133	**82**	92	8	139	101	54	36	99	31	**78**	**17**

9s (= 9a after transformation s) ≅ **9b**

12	67	53	137	6	44	91	109	46	114	63	**128**
49	**71**	**21**	**106**	**98**	**18**	**93**	**107**	**32**	**144**	**112**	**19**
102	**22**	**34**	136	48	5	35	85	135	**83**	**104**	81
86	94	72	**55**	20	79	4	29	**45**	129	138	119
56	24	134	88	**115**	70	105	**13**	15	77	131	42
3	143	118	50	84	**117**	**23**	25	65	58	76	108
142	2	27	95	61	**28**	**122**	120	80	87	69	37
89	121	11	57	**30**	75	40	**132**	130	68	14	103
59	51	73	**90**	125	66	141	116	**100**	16	7	26
43	**123**	**111**	9	97	140	110	60	10	**62**	**41**	64
96	**74**	**124**	**39**	**47**	**127**	**52**	**38**	**113**	**1**	**33**	**126**
133	78	92	8	139	101	54	36	99	31	82	**17**

9t (= 9a after transformation t) ≅ **9a**

12	**67**	53	137	6	44	91	109	46	114	**63**	**128**
49	**71**	**21**	**106**	**98**	**18**	**93**	**107**	**32**	**144**	**112**	**19**
102	**22**	**34**	136	48	5	35	85	135	**83**	**104**	81
86	**94**	72	**55**	20	79	4	29	**45**	129	**138**	119
56	**24**	134	88	**115**	70	105	**13**	15	77	**131**	42
3	**143**	118	50	84	**117**	**23**	25	65	58	**76**	108
142	**2**	27	95	61	**28**	**122**	120	80	87	**69**	37
89	**121**	11	57	**30**	75	40	**132**	130	68	**14**	103
59	**51**	73	**90**	125	66	141	116	**100**	16	**7**	26
43	**123**	**111**	9	97	140	110	60	10	**62**	**41**	64
96	**74**	**124**	**39**	**47**	**127**	**52**	**38**	**113**	**1**	**33**	**126**
133	**78**	92	8	139	101	54	36	99	31	**82**	**17**

On the squares 9a and 9b we can apply the transformation r also on the second 4-tuple and obtain 9c and 9d:
9a, 9b, 9c, 9d
With each of these 4 squares we can do transformation r with the third 4-tuple and get:
9e, 9f, 9g, 9h

All in all we obtain 8 essentially different axially symmetric trimagic squares of order 12.

9c

12	67	53	46	6	44	91	109	137	114	63	**128**
96	**33**	124	113	47	127	52	38	39	1	**74**	126
102	104	**34**	135	48	5	35	85	136	**83**	22	81
86	94	72	**55**	20	79	4	29	**45**	129	138	119
56	24	134	15	**115**	70	105	**13**	88	77	131	42
3	143	118	50	84	**117**	**23**	25	65	58	76	108
142	2	27	95	61	**28**	**122**	120	80	87	69	37
89	121	11	130	**30**	75	40	**132**	57	68	14	103
59	51	73	**90**	125	66	141	116	**100**	16	7	26
43	41	**111**	10	97	140	110	60	9	**62**	123	64
49	**112**	21	32	98	18	93	107	106	144	**71**	19
133	78	92	99	139	101	54	36	8	31	82	**17**

9d

12	63	53	46	6	44	91	109	137	114	67	**128**
96	**33**	124	113	47	127	52	38	39	1	**74**	126
102	104	**34**	135	48	5	35	85	136	**83**	22	81
86	138	72	**55**	20	79	4	29	**45**	129	94	119
56	131	134	15	**115**	70	105	**13**	88	77	24	42
3	76	118	50	84	**117**	**23**	25	65	58	143	108
142	69	27	95	61	**28**	**122**	120	80	87	2	37
89	14	11	130	**30**	75	40	**132**	57	68	121	103
59	7	73	**90**	125	66	141	116	**100**	16	51	26
43	41	**111**	10	97	140	110	60	9	**62**	123	64
49	**112**	21	32	98	18	93	107	106	144	**71**	19
133	82	92	99	139	101	54	36	8	31	78	**17**

9e

12	67	53	137	6	44	91	109	46	114	63	**128**
96	**33**	124	39	47	52	127	38	113	1	**74**	126
102	104	**34**	136	48	35	5	85	135	**83**	22	81
86	94	72	**55**	20	4	79	29	**45**	129	138	119
56	24	134	88	**115**	105	70	**13**	15	77	131	42
3	143	118	50	84	**117**	**23**	25	65	58	76	108
142	2	27	95	61	**28**	**122**	120	80	87	69	37
89	121	11	57	**30**	40	75	**132**	130	68	14	103
59	51	73	**90**	125	141	66	116	**100**	16	7	26
43	41	**111**	9	97	110	140	60	10	**62**	123	64
49	**112**	21	106	98	93	18	107	32	144	**71**	19
133	78	92	8	139	101	54	36	99	31	82	**17**

9f

12	63	53	137	6	44	91	109	46	114	67	**128**
96	**33**	124	39	47	52	127	38	113	1	**74**	126
102	104	**34**	136	48	35	5	85	135	**83**	22	81
86	138	72	**55**	20	4	79	29	**45**	129	94	119
56	131	134	88	**115**	105	70	**13**	15	77	24	42
3	76	118	50	84	**117**	**23**	25	65	58	143	108
142	69	27	95	61	**28**	**122**	120	80	87	2	37
89	14	11	57	**30**	40	75	**132**	130	68	121	103
59	7	73	**90**	125	141	66	116	**100**	16	51	26
43	41	**111**	9	97	110	140	60	10	**62**	123	64
49	**112**	21	106	98	93	18	107	32	144	**71**	19
133	82	92	8	139	101	54	36	99	31	78	**17**

9g

12	67	53	46	6	44	91	109	137	114	63	**128**
96	**33**	124	113	47	52	127	38	39	1	**74**	126
102	104	**34**	135	48	35	5	85	136	**83**	22	81
86	94	72	**55**	20	4	79	29	**45**	129	138	119
56	24	134	15	**115**	105	70	**13**	88	77	131	42
3	143	118	50	84	**117**	**23**	25	65	58	76	108
142	2	27	95	61	**28**	**122**	120	80	87	69	37
89	121	11	130	**30**	40	75	**132**	57	68	14	103
59	51	73	**90**	125	141	66	116	**100**	16	7	26
43	41	**111**	10	97	110	140	60	9	**62**	123	64
49	**112**	21	32	98	93	18	107	106	144	**71**	19
133	78	92	99	139	101	54	36	8	31	82	**17**

9h

12	63	53	46	6	44	91	109	137	114	67	**128**
96	**33**	124	113	47	52	127	38	39	1	**74**	126
102	104	**34**	135	48	35	5	85	136	**83**	22	81
86	138	72	**55**	20	4	79	29	**45**	129	94	119
56	131	134	15	**115**	105	70	**13**	88	77	24	42
3	76	118	50	84	**117**	**23**	25	65	58	143	108
142	69	27	95	61	**28**	**122**	120	80	87	2	37
89	14	11	130	**30**	40	75	**132**	57	68	121	103
59	7	73	**90**	125	141	66	116	**100**	16	51	26
43	41	**111**	10	97	110	140	60	9	**62**	123	64
49	**112**	21	32	98	93	18	107	106	144	**71**	19
133	82	92	99	139	101	54	36	8	31	78	**17**

The entries of both diagonals are equal in all 8 squares.
The entries of the 4-tuples are equal in all 8 squares.
The entries of columns 1, 3, 5, 8, 10 and 12 are equal in all 8 squares.

The differences between the squares can be seen for example in ...

Row 5

9a	56	24	134	88	**115**	70	105	**13**	15	77	131	42
9b	56	131	134	88	**115**	70	105	**13**	15	77	24	42
9c	56	24	134	15	**115**	70	105	**13**	88	77	131	42
9d	56	131	134	15	**115**	70	105	**13**	88	77	24	42
9e	56	24	134	88	**115**	105	70	**13**	15	77	131	42
9f	56	131	134	88	**115**	105	70	**13**	15	77	24	42
9g	56	24	134	15	**115**	105	70	**13**	88	77	131	42
9h	56	131	134	15	**115**	105	70	**13**	88	77	24	42